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Problem 11287. Players 1 through n play "continuous blackjack." At his turn, Player k considers a random number X_k drawn from the uniform distribution on $[0, 1]$. He may either accept X_k as his score or draw a second number Y_k from the same distribution, in which case his score is $X_k + Y_k$ if $X_k + Y_k < 1$ and 0 otherwise. The highest score wins. Give a rule for when Player k should draw a second number, in terms of k, n, the result of X_k, and the highest score attained so far.

Since the game is winner take all, Player k clearly must draw when the highest score so far exceeds X_k. If X_k is the highest score yet attained, then he should draw whenever $(2m + 1)X_k^{2m} + X_k^{2m+1} < 1$, where $m = n - k$ is the number of players who draw after Player k.

To see why, note that Player l who follows Player k has two chances to defeat him. First, there is a $1-X_k$ chance that $X_l > X_k$. Second, if $X_l < X_k$ then there is a $1-X_k$ chance that $X_k < X_l+Y_l < 1$. Thus there is a $1-X_k+X_k(1-X_k) = 1 - X_k^2$ chance that Player l will defeat Player k and a X_k^2 chance that Player k will defeat Player l. So if there are m players remaining, Player k has a X_k^{2m} chance of winning.

Now suppose Player k draws a second number Y_k. Define $S_k = X_k + Y_k$. Then it follows from the analysis in the last paragraph that Player k has a S_k^{2m} chance of winning if $S_k < 1$ and no chance of winning otherwise. S_k is drawn from the uniform distribution $[X_k, X_k + 1]$, so the chance that Player k will win after drawing a second number is $\int_{X_k}^{1} S^{2m} dS = \frac{1}{2m+1}(1 - X_k^{2m+1})$.

This means that the chances of winning are better by drawing a second number whenever $\frac{1}{2m+1}(1 - X_k)^{2m+1} > X_k^{2m}$, which can be rewritten as the expression in the first paragraph.